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SUB-SONIC FLOW ABOUT A SLENDER PROFILE IN A TUNNEL HAVING PERFORATED WALLS

[DOZVUKOVOE OBTEKANIE TONKOGO PROFILYA V KANALE S PERFORIROVANNYMI STENKAMI]

by

S. A. Glaskov

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AUTHORS' SUMMARY

A solution has been found for the problem of flow about a profile by an ideal incompressible liquid in a tunnel, the upper and lower walls of which may have varying porosity, depending on whether gas flows in or out of the tunnel. Calculations have been made of the distributions of pressure on the walls of the tunnel and on the profile located at an angle of attack on the axis of symmetry of the tunnel.

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INTRODUCTION

1 It is assumed that in the working section of the tunnel the flow is incompressible and irrotational or potential, and that the upper and lower perforated walls represent a tunnel of infinite length. The porosity coefficients of the upper and lower wall are f^+ and f^- respectively, outside the tunnel is surrounded by a cavity, in which the pressure is constant (Fig 1). The profile is situated on the mean line of the tunnel, the angle of inclination of the contour of the profile to the axis of the abscissa $h^{\pm}(x)$ are considered to be small: ($h^+(x)$ – for the upper part of the profile, and $h^-(x)$ – for the lower part).

The region D in which a solution of the problem is sought, represents the interior of a two-dimensional tunnel having a width H, with a slit along the interval [0, 1], where a slender lifting profile is located.

In a linear formulation, by transferring the boundary conditions to the axis x, we obtain

$$V(x, +0) = h^+(x), \quad V(x, -0) = h^-(x) , \tag{1}$$

where V is the perturbed vertical component of velocity, referred to the velocity of the incident flow. The conditions on the walls

$$\left.\begin{aligned} y &= \frac{H}{2} : x \in (-\infty, x), \quad V = R_1^+U; \quad x \in (x_1, \infty), \quad V = -R_2^+U; \\ y &= -\frac{H}{2} : x \in (-\infty, x_2), \quad V = R_2^-U; \quad x \in (x_2; \infty), \quad V = R_1^-U , \end{aligned}\right\} \tag{2}$$

where U is the perturbed horizontal component of the component of velocity, referred to the velocity of the incident flow, x_1 and x_2 are the coordinates of the points at which the porosity is equal to 0. R_1^+, R_2^+, R_1^-, R_2^- are the parameters of porosity dependent upon the coefficient of porosity f^+ and f^-, and also on whether the gas flows into or out of the tunnel. Fig 2 gives the results of the investigation of the porosity of porous panels in Ref 4.

The solution of the problem posed is reduced to finding the analytical function $\phi(z) = U - iV$ of the complex variable $z = x + iy$ in the region D which satisfies the boundary conditions (1) and (2).

2 Let us break down the procedure for obtaining a solution into several stages:

(a) Finding in the region D an analytical function $L(2) = L_R - iL_I$ which satisfies the boundary condition (2) solely on the lines $y^{\pm} = \pm\frac{H}{2}$ while being continuous across the section $x \in [0, 1]$, $y = 0$.

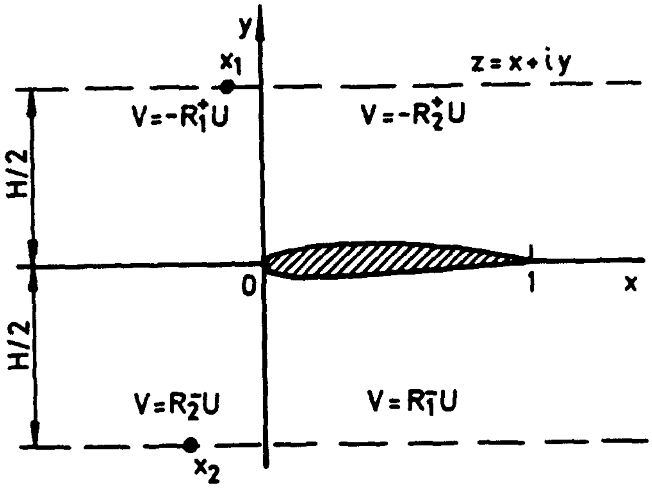

Fig 1

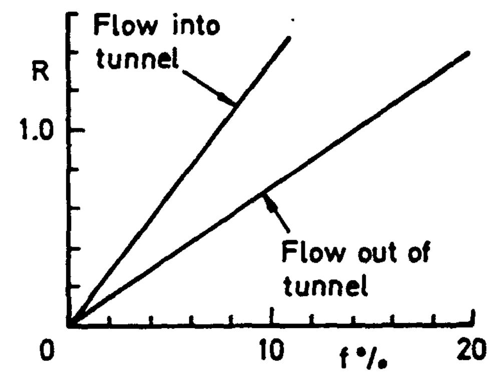

Fig 2

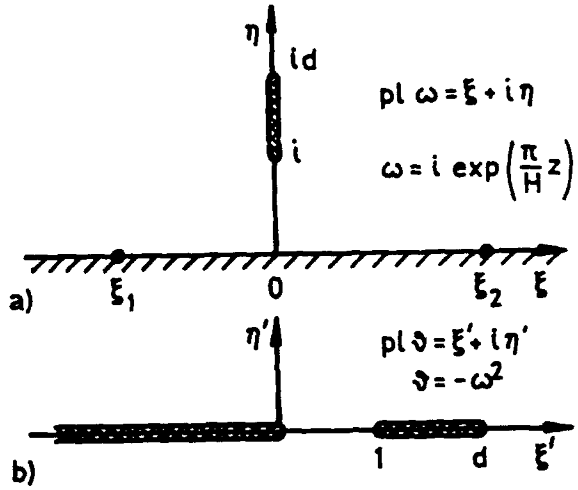

Fig 3

Let us transfer the strip in the plane 2, bounded by the lines $y^{\pm} = \pm \frac{H}{2}$, by the conformal transformation $w = i \exp\left(\frac{\pi}{H} z\right)$, into the upper half-plane $w = \xi + i\eta$ (Fig 3a). For $L(w(z))$ the conditions (2) on the true axis in the plane w assume the form

$$\eta = 0 \begin{cases} \xi \in (-\infty, \xi_1(x_1)), & L_1 = -R_2^+ L_R\,; & (3.1) \\ \xi \in (\xi_1(x_1), 0), & L_1 = -R_1^+ L_R\,; & (3.2) \\ \xi \in (0, \xi_2(x_2)), & L_1 = R_1^- L_R\,; & (3.3) \\ \xi \in (\xi_2(x_2), \infty), & L_1 = R_2^- L_R\,. & (3.4) \end{cases}$$

Let us write the analytical function $L(w)$ in a general form:

$$L(w) = (w-\xi_1)^{\gamma_3}(w)^{\gamma_2}(w-\xi_2)^{\gamma_1}\frac{P(w)}{Q(w)}a \ ,$$

where a is a complex constant, $P(w)$ and $Q(w)$ are arbitrary polynomials with real coefficients which are independent of conditions (3.1 to 3.4), and $\gamma_1, \gamma_2, \gamma_3$ are real constants.

Let us determine

$$L(w) = L_R - iL_I = |\xi-\xi_1|^{\gamma_3}|\xi|^{\gamma_2}|\xi-\xi_2|^{\gamma_1}\frac{P(\xi)}{Q(\xi)}a \ ,$$

from the condition (3.4)

$$a = \exp\left(-i \operatorname{arctg} R_2^-\right); \quad \text{for } \xi \in (0, \xi_2), \ \eta = 0$$

$$L(w) = L_R - iL_I = |\xi-\xi_1|^{\gamma_3}|\xi|^{\gamma_2}|\xi-\xi_2|^{\gamma_1}\frac{P(\xi)}{Q(\xi)}\exp\left(i\pi\gamma_1 + i \operatorname{arctg} R_2^-\right) \ ,$$

from the condition (3.3)

$$\gamma_1 = \frac{1}{\pi}\left(\operatorname{arctg} R_2^- - \operatorname{arctg} R_1^-\right) + n_1 , \quad \text{where } n_1 = 1, 0; \quad \text{for } \xi \in (\xi_1, 0) ,$$

$$\eta = 0$$

$$L(w) = L_R - iL_I = |\xi-\xi_1|^{\gamma_3}|\xi|^{\gamma_2}|\xi-\xi_2|^{\gamma_1}\frac{P(\xi)}{Q(\xi)}\exp\left(i\pi\gamma_1 + i\pi\gamma_2 + \right.$$

$$\left. + i \operatorname{arctg} R_2^-\right) \ ,$$

From the condition (3.2)

$$\gamma_2 = \frac{1}{\pi}\left(\operatorname{arctg} R_1^+ + \operatorname{arctg} R_1^-\right) + n_2 , \quad \text{where } n_2 = -1, 0; \quad \text{for } \xi \in (-\infty, \xi_1) ,$$

$$\eta = 0$$

$$L(w) = -L_R - iL_I = |\xi-\xi_1|^{\gamma_3}|\xi|^{\gamma_2}|\xi-\xi_2|^{\gamma_1}\frac{P(\xi)}{Q(\xi)}\exp\left(i\pi\gamma_1 + i\pi\gamma_2 + i\pi\gamma_3 + \right.$$

$$\left. + i \operatorname{arctg} R_2^-\right) \ ,$$

From the condition (3.1)

$$\gamma_3 = \frac{1}{\pi}\left(\operatorname{arctg} R_2^+ - \operatorname{arctg} R_1^+\right) + n_3 , \quad \text{where } n_3 = -1, 0;$$

$n_1 = n_2 = n_3 = 0$ from the condition of smoothness of the solution in the case of non-perforated walls ($R = 0$).

On the section $\xi' \in [1, d]$, $\eta_1 = 0$, $d = \exp\left(\frac{2\pi}{H}\right)$ in the plane $\zeta = \xi' + i\eta'$ (see Fig 3b) the function $L(\zeta(w(z)))$ in the general case has a complex value

$$L(\zeta(w(z))) = |L| \exp(i\varphi_L) , \tag{4}$$

where $\varphi_L = \varphi_{L_0} + \varphi_*$,

$$\varphi_{L_0} = \mathrm{ARG}\left[(i\sqrt{\xi'} - \xi_1)^{\gamma_3}\right] + \mathrm{ARG}\left[(i\sqrt{\xi'})^{\gamma_2}\right] + \mathrm{ARG}\left[(i\sqrt{\xi'} - \zeta_2)^{\gamma_1}\right] ,$$

$$\varphi_* = \mathrm{ARG}\left[\frac{P(i\sqrt{\xi'})}{Q(i\sqrt{\xi'})}\right] ;$$

(b) Let us find an analytical function $F(\zeta)$ such, that in the plane $\zeta = \exp\left(\frac{2\pi}{H} z\right) = -w^2 = \xi' + i\eta'$, having two cuts along the real axis $\xi' \in (-\infty, 0)$ and $\xi' \in \left[1, d = \exp\left(\frac{2\pi}{H}\right)\right]$, it satisfies the following

$$\begin{aligned} &\xi' \in (-\infty, 0)\ \eta' = \pm 0, \mathrm{Im}\, F = 0 , \\ &\xi' \in [1, d] \quad \eta' = \pm 0, \mathrm{Im}\, F = -\varphi_L . \end{aligned} \tag{5}$$

In general form the function $F(\zeta)$ which satisfies condition (5), assumes the form:

$$F(\zeta) = \frac{1}{\pi} \zeta^{\frac{1}{2}-k_1} (\zeta - 1)^{\frac{1}{2}-k_2} (d - \zeta)^{\frac{1}{2}-k_3} \left(\int_1^d \varphi_L(t) \frac{t^{k_1-\frac{1}{2}} (t-1)^{k_2-\frac{1}{2}} (d-t)^{k_3-\frac{1}{2}}}{t - \zeta} dt + p_0 + p_1 \zeta \right) ,$$

where $k_1 = 0, 1$; $k_2 = 0, 1$; $k_3 = 0, 1$; $p_0 = \mathrm{const}$, $p_1 = \mathrm{const}$.

From this it will be seen that for no values of k_j can the condition of boundedness at all the points indicated be satisfied, as a minimum at one point of the four indicated is not limited, unless an additional condition is imposed on φ_L for $k_1 + k_2 + k_3 = \begin{cases} 1 \\ 0 \end{cases}, p_0 = p_1 = 0$.

$$-\int_1^d \varphi_*(t)\frac{dt}{\sqrt{t}\sqrt{d-t}\sqrt{t-1}} = \int_1^d \varphi_{L_0}(t)\frac{dt}{\sqrt{t}\sqrt{d-t}\sqrt{t-1}} . \tag{6}$$

Taking this into account we demonstrate that with the condition (6) for the integer positive values k_1, k_2, k_3, such that $k_1+k_2+k_3=\left\{\begin{smallmatrix}0\\1\end{smallmatrix}\right.$, $p_0=p_1=0$, the following equation is satisfied

$$\sqrt{\zeta}\sqrt{\zeta-1}\sqrt{d-\zeta}\int_1^d(\varphi_{L_0}+\varphi_*)\frac{dt}{\sqrt{t}\sqrt{d-t}\sqrt{t-1}\,(t-\zeta)} =$$

$$= \zeta^{\frac{1}{2}-k_1}(\zeta-1)^{\frac{1}{2}-k_2}(d-\zeta)^{\frac{1}{2}-k_3}\int_1^d(\varphi_{L_0}+\varphi_*)\,t^{k_1-\frac{1}{2}}(t-1)^{k_2-\frac{1}{2}}(d-t)^{k_3-\frac{1}{2}}\frac{dt}{t-\zeta} . \tag{7}$$

It is sufficient to prove the correctness of equation (7) for the case $k_1=k_2=0$, $k_3=1$, since the remaining ones are similar.

Let us examine the difference of the left-hand and right-hand parts in expression (7):

$$\sqrt{\zeta}\sqrt{\zeta-1}\left\{\sqrt{d-\zeta}\int_1^d(\varphi_{L_0}+\varphi_*)\frac{dt}{\sqrt{t}\sqrt{t-1}\sqrt{d+t}\,(t-\zeta)} -\right.$$

$$\left.-\frac{1}{\sqrt{d-\zeta}}\int_1^d\frac{(\varphi_{L_0}+\varphi_*)\sqrt{d-t}\,dt}{\sqrt{t}\sqrt{t-1}\,(t-\zeta)}\right\} = \sqrt{\zeta}\sqrt{\zeta-1}\int_1^d(\varphi_{L_0}+\varphi_*)\times$$

$$\times\left(\sqrt{\frac{d-\zeta}{d-t}}-\sqrt{\frac{d-t}{d-\zeta}}\right)\frac{dt}{\sqrt{t}\sqrt{t-1}\,(t-\zeta)} = \sqrt{\zeta}\sqrt{\zeta-1}\frac{1}{\sqrt{d-\zeta}}\int_1^d(\varphi_{L_0}+\varphi_*)\times$$

$$\times\frac{dt}{\sqrt{t}\sqrt{t-1}\sqrt{d-t}} = \sqrt{\zeta}\sqrt{\zeta-1}\frac{1}{\sqrt{d-\zeta}}.0 = 0 ,$$

which in fact had to be demonstrated. Thus, on the condition (6), the required analytical function assumes the form

$$F(\zeta) = \frac{1}{\pi}\sqrt{\zeta}\sqrt{\zeta-1}\sqrt{d-\zeta}\int_1^d (\varphi_{L_0}+\varphi_*)\frac{dt}{\sqrt{t}\sqrt{t-1}\sqrt{d-t}\,(t-\zeta)}\ ;$$

(c) Let us examine the analytical function $T(\zeta) = L(\zeta)\exp(F(\zeta))$. The function $T(\zeta)$ has discontinuities along the line $\xi' \in (-\infty, 0)$, $\eta' = 0$ and on the section $\xi' \in [1, d]$, $\eta' = 0$. It is easy to see: $T(\zeta) = T_R - iT_1$ satisfies conditions on the line $\xi' \in (-\infty, 0)$, $\eta' = 0$ which are analogous to the boundary condition (3) for $L(\zeta)$, while on the section $\xi' \in [1, d]$, $\eta' = 0$ the function $T(\zeta)$ is real.

Let us introduce a further subsidiary function $\Phi' = \Phi/T$; then taking into account stages (a) and (b) for Φ' the boundary conditions in the plane ζ assume the form:

$$\left.\begin{aligned} \eta' &= \pm 0,\quad \xi' \in (-\infty, 0),\quad \operatorname{Im}\Phi'^{\pm} = 0\ ;\\ \eta' &= \pm 0,\quad \xi' \in [1, d],\quad \operatorname{Im}\Phi'^{\pm} = -h^{\pm}(\xi)/T^{\pm}(\xi)\ , \end{aligned}\right\} \tag{8}$$

where the sign «+» corresponds to the upper edge of the slit, and the sign «–» corresponds to the lower one.

Let us represent the analytical function Φ' as the sum of the symmetrical and anti-symmetrical analytical functions: $\Phi' = \Phi'_S + \Phi'_A$ such, that

$$\Phi'_A(\xi', \eta') = U_A - iV_A,\quad U_A(\xi', \eta') = -U_A(\xi', -\eta')\ ,$$

$$V_A(\xi', \eta') = V_A(\xi', -\eta')\ ;$$

$$\Phi'_S(\xi', \eta') = U_S - iV_S,\quad V_S(\xi', \eta') = -V_S(\xi', -\eta')\ ,$$

$$U_S(\xi', \eta') = U_S(\xi', -\eta')\ .$$

Thus for Q'_A and Q'_S condition (8) will give

$$\left\{\begin{aligned} &\eta' = \pm 0,\quad \xi' \in (-\infty, 0),\quad \operatorname{Im}\Phi'^{\pm}_A = 0\ ;\\ &\eta' = \pm 0,\quad \xi' \in [1, d],\quad \operatorname{Im}\Phi'^{\pm}_A = \left(\frac{h^+}{T^+} + \frac{h^-}{T^-}\right)\tfrac{1}{2}\ ; \end{aligned}\right.$$

$$\begin{cases} \eta' = \pm 0\,, \quad \xi' \in (-\infty, 0)\,, \quad \operatorname{Im} \Phi'_S \pm = 0\ ; \\[2ex] \eta' = \pm 0\,, \quad \xi' \in [1, d]\,, \quad \operatorname{Im} \Phi'_S \pm = \left(\dfrac{h^+}{T^+} + \dfrac{h^-}{T^-}\right) \tfrac{1}{2}\ . \end{cases}$$

Taking into account the Zhukovskii condition on the trailing edge of the profile and the boundedness of the function to infinity, in accordance with [7], we obtain

$$\Phi'_A = -\frac{1}{2\pi}\sqrt{\xi}\sqrt{\frac{d-\zeta}{\zeta-1}}\int_1^d \left(\frac{h^+}{T^+} + \frac{h^-}{T^-}\right)\sqrt{\frac{t-1}{d-t}}\,\frac{dt}{t-\zeta}\ ;$$

$$\Phi'_S = -\frac{1}{2\pi}\int_1^d \left(\frac{h^+}{T^+} - \frac{h^-}{T^-}\right)\frac{dt}{t-\zeta} + M_n(\zeta)\ .$$

The general form of the solution for Φ assumes the form

$$\Phi = T\Phi' = \frac{-1}{2\pi}T(\zeta)\left\{\sqrt{\zeta}\sqrt{\frac{d-t}{t-1}}\int_1^d \left(\frac{h^+}{T^+} + \frac{h^-}{T^-}\right)\sqrt{\frac{t-1}{d-t}}\,\frac{dt}{t-\zeta} + \right.$$

$$\left. + \int_1^d \left(\frac{h^+}{T^+} - \frac{h^-}{T^-}\right)\frac{dt}{t-\zeta} + M_n(\zeta)\right\}\ ,$$

where $M_n(\zeta)$ is a polynomial with real coefficients. It is easy to see that the limit solution ($\Phi \to 0\,,\ x \to \pm\infty$) may be obtained solely on condition that $\Gamma(w)/Q(w) = (Ai\sqrt{\zeta} + 1)^{-1}$ and $M_n(\zeta) = c_0 = \text{const}$. The magnitude of the constant c_0 is determined from the condition of absence of a pole at the point $\zeta = -\sqrt{A}$.

The associated complex velocity may be written in the following form:

$$\Phi(\zeta(z)) = U - iV = \frac{1}{2\pi}(i\sqrt{\zeta}-\xi_1)^{\gamma_3}(i\sqrt{\zeta})^{\gamma_2}(i\sqrt{\zeta}-\xi_2)^{\gamma_1}\frac{\exp\left(i\,\mathrm{arctg}\,R_2^-\right)}{1+i\sqrt{\zeta}A}\times$$

$$\times\exp\left[-\frac{1}{\pi}\sqrt{\zeta}\sqrt{d-\zeta}\sqrt{\zeta-1}\int_1^d(\varphi_{L_0}+\varphi_*)\frac{dt}{\sqrt{t}\sqrt{d-t}\sqrt{t-1}\,(t-\zeta)}\right]\times$$

$$\times\left\{\int_1^d\frac{h^+/\exp(F^+)-h^-/\exp(F^-)}{|L(t)|\exp\left(i\left(\varphi_{L_0}+\varphi_*\right)\right)(t-\zeta)}dt+\right.$$

$$\left.+\sqrt{\zeta}\sqrt{\frac{d-\zeta}{\zeta-1}}\int_1^d\frac{\left\{h^+/\exp(F^+)+h^-/\exp(F^-)\right\}\sqrt{t-1}}{|L(t)|\exp\left(i\left(\varphi_{L_0}+\varphi_*\right)\right)\sqrt{t}\sqrt{d-t}\,(t-\zeta)}dt+c_0\right\},$$

where $\zeta = \exp\left(\frac{2\pi}{H}z\right)$, $\xi_1 = -\exp\left(\frac{\pi}{H}x_1\right)$, $\xi_2 = -\exp\left(\frac{\pi}{H}x_2\right)$,

$$F^{\pm} = \pm\frac{1}{\pi}\sqrt{t}\sqrt{t-1}\sqrt{d-t}\int_1^d\frac{\left(\varphi_{L_0}+\varphi_*\right)ds}{\sqrt{s}\sqrt{s-1}\sqrt{d-s}\,(s-t)}-i\left(\varphi_{L_0}+\varphi_*\right)=$$

$$= \pm F_0 - i\left(\varphi_{L_0}+\varphi_*\right),\quad t\in[1,d],$$

for the following conditions:

$$\int_1^d\left(\varphi_{L_0}+\varphi_*\right)\frac{dt}{\sqrt{t}\sqrt{d-t}\sqrt{t-1}} = 0,\quad \varphi_* = -\mathrm{arctg}\,(A\sqrt{t});\tag{9.1}$$

$$\int_1^d\frac{h^+/\exp(-F_0)-h^-/\exp(F_0)}{|L(t)|\left(t+\sqrt{|A|^{-1}}\right)}dt+\mathrm{sign}\,(A)\times$$

$$\times|A|^{-1/4}\sqrt{\frac{d+\sqrt{|A|^{-1}}}{\sqrt{|A|^{-1}}+1}}\int_1^d\frac{\left(h^+/\exp(F-F_0)h^-/\exp(F_0)\right)\sqrt{t-1}}{|L(t)|\sqrt{t}\sqrt{d-t}\left(t+\sqrt{|A\Gamma|^{-1}}\right)}dt+c_0 = 0,\tag{9.2}$$

$$\int_1^d \frac{h^+/\exp(-F_0) - h^-/\exp(F_0)}{|L(t)|\left(t+\xi_1^2\right)}\,dt +$$

$$+\xi_1\sqrt{\frac{d+\xi_1^2}{1+\xi_1^2}}\int_1^d \frac{h^+/\exp(-F_0) + h^-/\exp(F_0)}{|L(t)|\sqrt{t}\sqrt{d-t}\left(t+\xi_1^2\right)}\sqrt{t-1}\,dt + c_0 = 0\ , \qquad (9.3)$$

$$\int_1^d \frac{h^+/\exp(-F_0) - h^-/\exp(F_0)}{|L(t)|\left(\xi_2^2\right)}\,dt -$$

$$+\xi_2\sqrt{\frac{d+\xi_2^2}{1+\xi_2^2}}\int_1^d \frac{h^+/\exp(-F_0) + h^-/\exp(F_0)}{|L(t)|\sqrt{t}\sqrt{d-t}\left(t+\xi_2^2\right)}\sqrt{t-1}\,dt + c_0 = 0\ . \qquad (9.4)$$

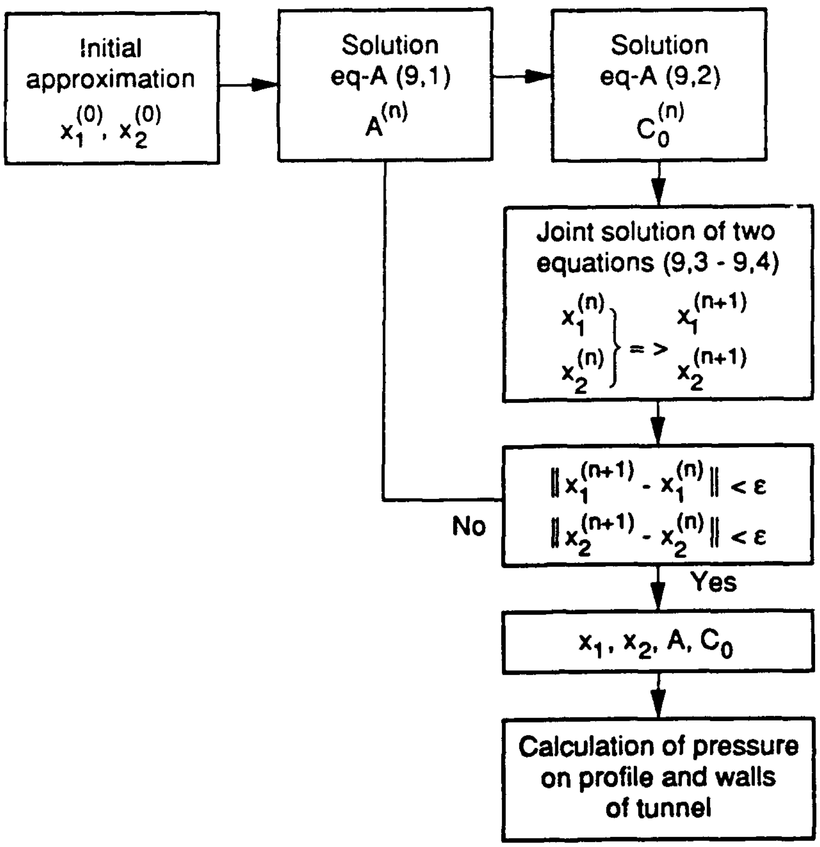

Fig 4

Conditions (9.3) and (9.4) for the determination of x_1 and x_2 automatically follow from the circumstance that during passage along the wall through the points x_1 and x_2 the pressure coefficient and the vertical component of velocity change their sign.

In essence, solution of the problem is reduced to determining the four constants A, c_0, x_1, x_2. A diagram of the solution of the system of equations (9.1) to (9.4) is given by Fig 4.

The pressure coefficient on the porous boundary is equal to

$$c_p = -2U = \frac{1}{\pi} G \left| L\left(\zeta\left(x \pm i\frac{H}{2}\right)\right)\right| \times$$

$$\times \exp\left(\pm\sqrt{|\zeta|}\sqrt{d-\zeta}\sqrt{|\zeta-1|}\int_1^d \frac{(\varphi_{L_0}+\varphi_*)\,dt}{\sqrt{t}\sqrt{d-t}\sqrt{t-1}(t-\zeta)}\right)\times$$

$$\times\left\{\int_1^d \frac{h^+/\exp(-F_0) - h^-/\exp(F_0)}{|L(t)|(t-\zeta)}\,dt \pm\right.$$

$$\left.\pm\sqrt{|\zeta|}\sqrt{\frac{d-\zeta}{|\zeta-1|}}\int_1^d \frac{h^+/\exp(-F_0) - h^-/\exp(F_0)}{|L(t)|\sqrt{t}\sqrt{d-t}(t-\zeta)}\sqrt{t-1}\,dt + c_0\right\},$$

where

$$G = \begin{cases} \cos\left(-\operatorname{arctg} R_2^+\right) \\ \cos\left(-\operatorname{arctg} R_2^+ + \pi\gamma_1\right) \\ \cos\left(-\operatorname{arctg} R_2^+ + \pi(\gamma_1+\gamma_2)\right) \\ \cos\left(-\operatorname{arctg} R_2^+ + \pi(\gamma_1+\gamma_2+\gamma_3)\right) \end{cases}$$

$$\zeta = \quad \zeta\left(x - i\frac{H}{2}\right)^{x>x_2}_{x<x_2} \quad \text{Lower wall}$$

$$\zeta = \quad \zeta\left(x + i\frac{H}{2}\right)^{x<x_1}_{x>x_1} \quad \text{Upper wall}$$

The pressure coefficient on the profile is equal to

$$c_p^{\pm} = -2U^{\pm} = \frac{1}{\pi}\left|L\big(\zeta(x+i0)\big)\right| \times$$

$$\times \exp^{-1}\left(\pm\sqrt{\zeta}\sqrt{d-\zeta}\sqrt{\zeta-1}\int_1^d \frac{(\varphi_{L_0}+\varphi_*)\,dt}{\sqrt{t}\sqrt{d-t}\sqrt{t-1}\,(t-\zeta)}\right)\times$$

$$\times \int_1^d \frac{h^+/\exp(-F_0) - h^-/\exp(F_0)}{|L(t)|\sqrt{t}\sqrt{d-t}\,(t-\zeta)}\,dt \pm$$

$$\pm\sqrt{\zeta}\sqrt{\frac{d-\zeta}{\zeta-1}}\int_1^d \frac{h^+/\exp(-F_0) - h^-/\exp(F_0)}{|L(t)|\sqrt{t}\sqrt{d-t}\,(t-\zeta)}\sqrt{t-1}\,dt + c_0 \quad ,$$

where the sign «+» denotes the upper surface of the profile, and «–» the lower.

3 Calculations of the pressure on the profile, which were performed for comparison in the simplest case (symmetrical problem of boundary conditions on the upper and lower walls, or $H \to \infty$), gave a good agreement with the results of the calculation made in Refs 1, 2 and 5.

Figs 5 and 6 show calculations of flow about a plate at an angle of attack of 3° in a tunnel ($H = 6$) dependent upon boundary conditions in porous walls.

As follows from Refs 3 and 4, symmetrical assigning on the upper and lower walls of the tunnel of the parameter of porosity R corresponds to a differing coefficient of porosity of these walls.

As a rule, in wind tunnels, the coefficient of porosity is identical for the upper and lower wall. As will be seen from the calculations performed, the non-symmetrical assigning of the boundary conditions on the porous surface perceptibly changes the flow picture. A change over the length of the tunnel of the parameter of porosity on the upper wall has a slight effect on the distribution of pressure on the boundary, while it is practically absent in the region of location of the plate.

Therefore, in the case of an identical coefficient of porosity of the walls it is necessary to take into account the difference in the parameter of porosity dependent upon the direction of flow of the gas, that is, into the tunnel or out of it.

The author wishes to thank V.M. Neiland and O.K. Semenov for useful discussions.

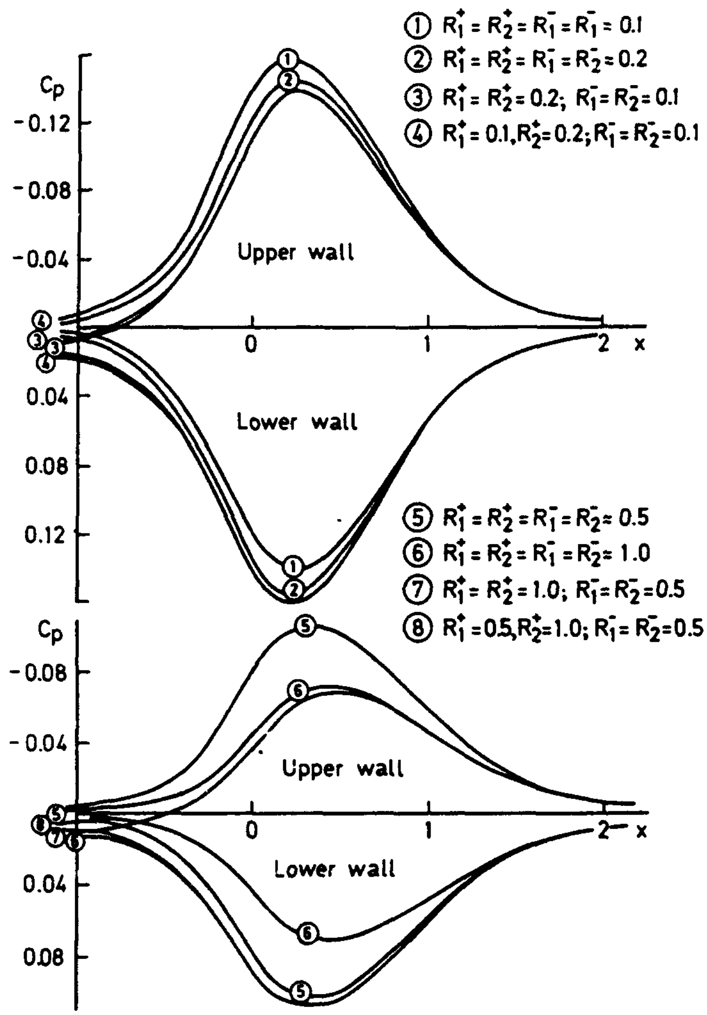
① $R_1^+ = R_2^+ = R_1^- = R_1^- = 0.1$
② $R_1^+ = R_2^+ = R_1^- = R_2^- = 0.2$
③ $R_1^+ = R_2^+ = 0.2$; $R_1^- = R_2^- = 0.1$
④ $R_1^+ = 0.1, R_2^+ = 0.2$; $R_1^- = R_2^- = 0.1$
Cp
-0.12
-0.08
-0.04
Upper wall
0
1
2 x
0.04
Lower wall
0.08
0.12
⑤ $R_1^+ = R_2^+ = R_1^- = R_2^- = 0.5$
⑥ $R_1^+ = R_2^+ = R_1^- = R_2^- = 1.0$
⑦ $R_1^+ = R_2^+ = 1.0$; $R_1^- = R_2^- = 0.5$
⑧ $R_1^+ = 0.5, R_2^+ = 1.0$; $R_1^- = R_2^- = 0.5$
Cp
-0.08
-0.04
Upper wall
0
1
2 x
Lower wall
0.04
0.08

Fig 5

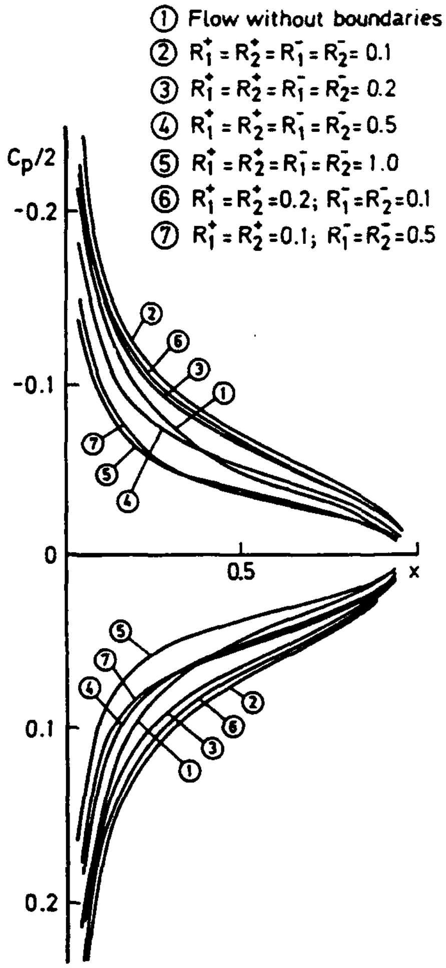
① Flow without boundaries
② $R_1^+ = R_2^+ = R_1^- = R_2^- = 0.1$
③ $R_1^+ = R_2^+ = R_1^- = R_2^- = 0.2$
④ $R_1^+ = R_2^+ = R_1^- = R_2^- = 0.5$
⑤ $R_1^+ = R_2^+ = R_1^- = R_2^- = 1.0$
⑥ $R_1^+ = R_2^+ = 0.2$; $R_1^- = R_2^- = 0.1$
⑦ $R_1^+ = R_2^+ = 0.1$; $R_1^- = R_2^- = 0.5$
Cp/2
-0.2
-0.1
0
0.5
x
0.1
0.2

Fig 6

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